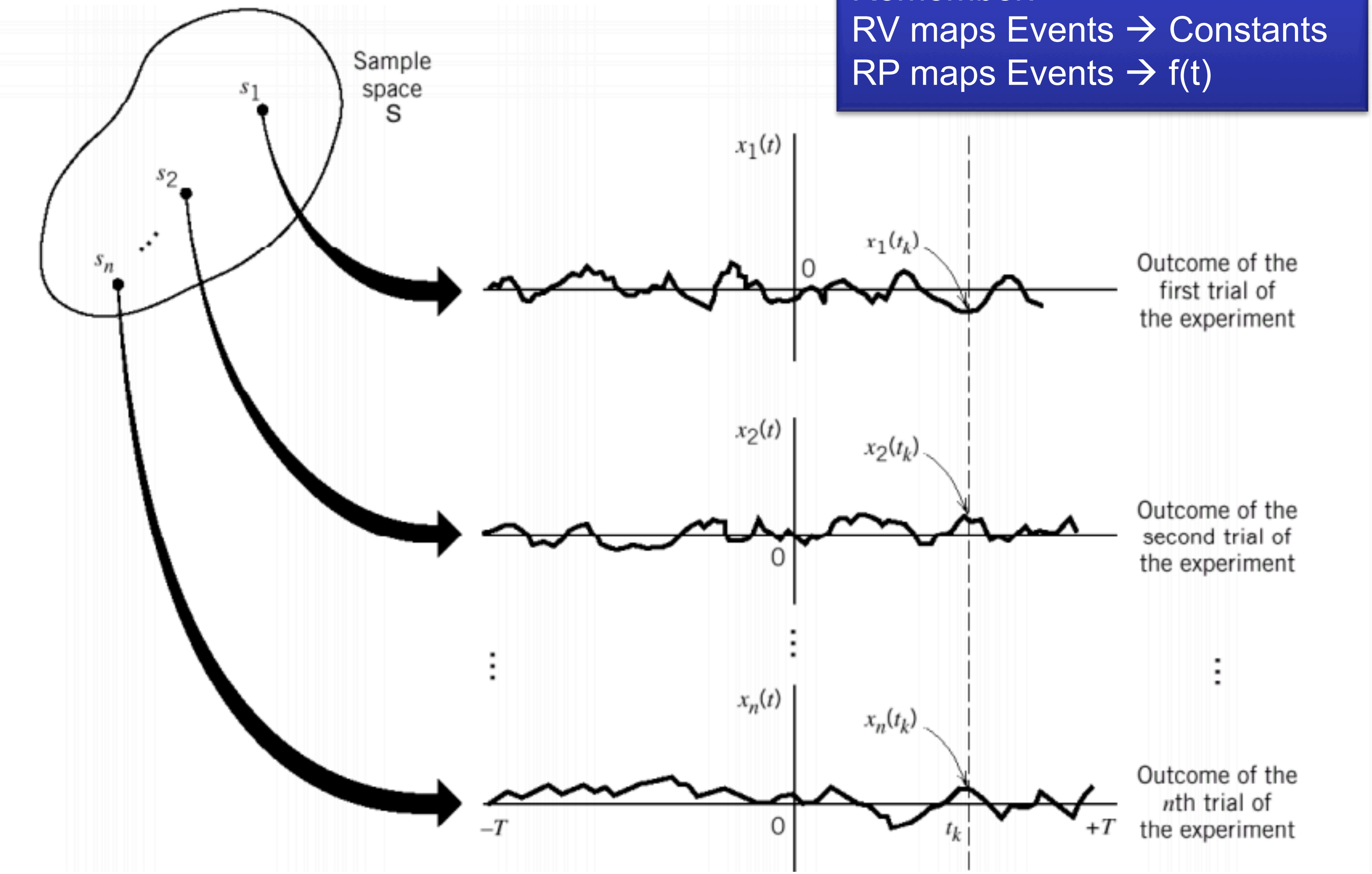
Random Process

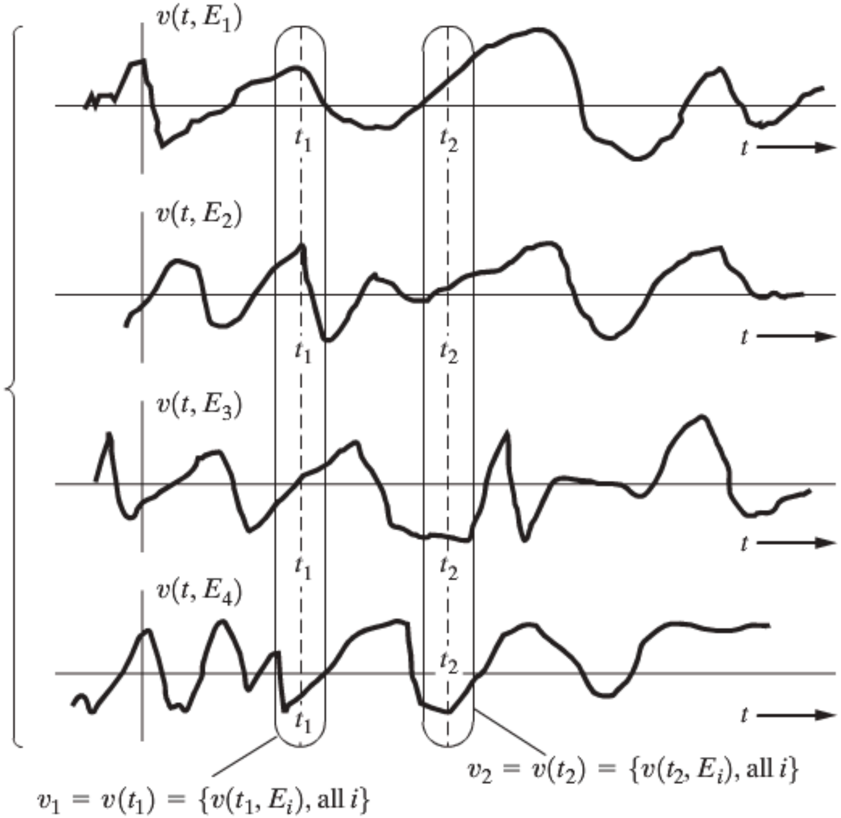
- A random process is a time-varying function that assigns the outcome of a random experiment to each time instant: X(t).
- For a fixed (sample path): a random process is a time varying function, e.g., a signal.
  - For fixed t: a random process is a random variable.
- If one scans all possible outcomes of the underlying random experiment, we shall get an ensemble of signals.
- Random Process can be **continuous or discrete**
- Real random process also called stochastic process
  - Example: Noise source (Noise can often be modeled as a Gaussian random process.

An Ensemble of Signals

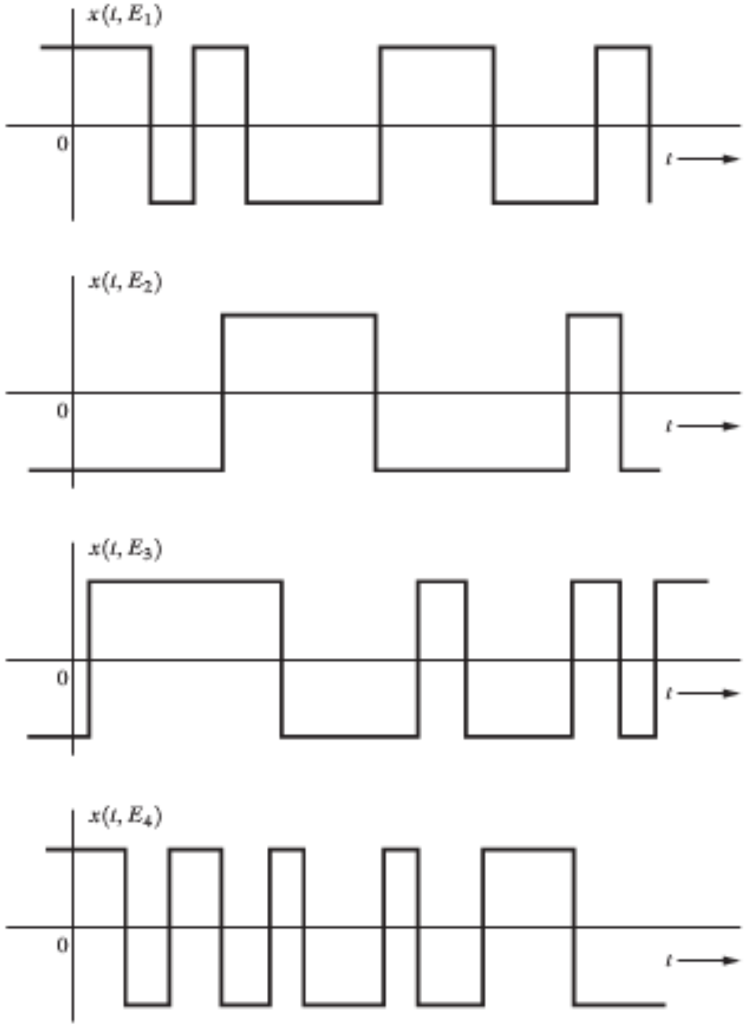
Remember:
RV maps Events → Constants
RP maps Events → f(t)



RP: Discrete and Continuous



The set of all possible sample functions {v(t, E i)} is called the ensemble and defines the random process v(t) that describes the noise source.



Sample functions of a binary random process.

RP Characterization

- Random variables x 1 , x 2 , . . . , x n represent amplitudes of sample functions at t 5 t 1 , t 2 , . . . , t n .
  - A random process can, therefore, be viewed as a collection of an infinite number of random variables:

$$\text{joint PDF } f_x(x_1, x_2, \ldots, x_n, t_1, t_2, \ldots, t_n) :$$

RP Characterization – First Order

- CDF

$$F_x(x, t) = P\{\mathbf{x}(t) \leq x\}$$

- PDF

$$\boxed{f_x(x, t)} = \frac{dF_x(x, t)}{dx}$$

- Mean

$$m_x(t) = \overline{\mathbf{x}(t)} = E\{\mathbf{x}(t)\} = \int_{-\infty}^{+\infty} x f_x(x, t)\,dx$$

- Mean-Square

$$\overline{\mathbf{x}^2(t)} = E\{\mathbf{x}^2(t)\} = \int_{-\infty}^{+\infty} x^2 f_x(x, t)\,dx$$

Statistics of a Random Process

- For fixed *t*: the random process becomes a random variable, with mean

$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x\, f_X(x;t)dx$$

  – In general, the mean is a function of t.

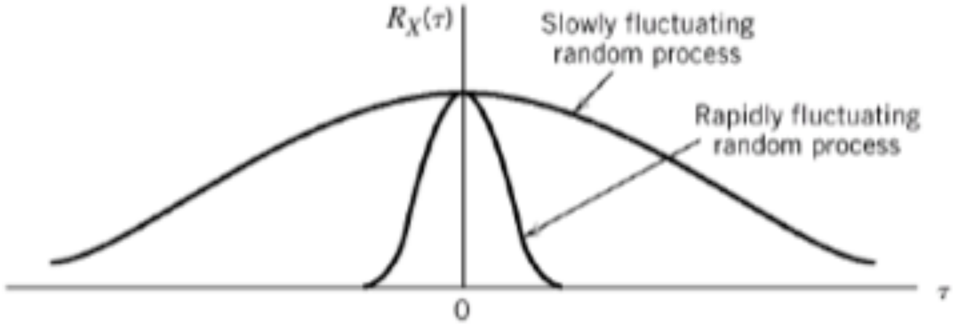
- Autocorrelation function

$$R_X(t_1,t_2) = E[X(t_1)X(t_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_X(x,y;t_1,t_2)dxdy$$

  – In general, the autocorrelation function is a two-variable function.
  – It measures the correlation between two samples.

RP Characterization – Second Order

- The first order does not provide sufficient information as to how rapidly the RP is changing as a function of time→ We use second order estimation



RP Characterization – Second Order

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$$F_x(x_1, x_2, t_1, t_2) = P\{x(t_1) \le x_1, x(t_2) \le x_2\}$$

- CDF

$$f_x(x_1, x_2, t_1, t_2) = \frac{\partial^2 F_x(x_1, x_2, t_1, t_2)}{\partial x_1 \partial x_2}$$

- PDF

- Auto-correlation

$$R_x(t_1, t_2) = E\{x(t_1)x(t_2)\}$$

(statistical average of the product of RVs)

$$R_{xy}(t_1, t_2) = E\{x(t_1)y(t_2)\}$$

- Cross-Correlation

(measure of correlation between sample function amplitudes of processes x (t) and y (t) at time instants t 1 and t 2 , respectively)