What does Linear Programming (LP) mean?

Linear programming is a mathematical method that is used to determine the best possible outcome or solution from a given set of parameters or list of requirements, which are represented in the form of linear relationships. It is most often used in computer modeling or simulation in order to find the best solution in allocating finite resources such as money, energy, manpower, machine resources, time, space and many other variables. In most cases, the "best outcome" needed from linear programming is maximum profit or lowest cost or loss.

Because of its nature, linear programming is also called 'linear optimization'.

Linear programming is used as a mathematical method for determining and planning for the best outcomes and was developed during World War II by Leonid Kantorovich in 1937. It was a method used to plan expenditures and returns in a way that reduced costs for the military and possibly caused the opposite for the enemy.

Linear programming is part of an important area of mathematics called "optimization techniques" as it is literally used to find the most optimized solution to a given problem. A very basic example of linear optimization usage is in logistics or the "method of moving things around efficiently." For example, suppose there are 1000 boxes of the same size of 1 cubic meter each; 3 trucks that are able to carry 100 boxes, 70 boxes and 40 boxes respectively; several possible routes; and 48 hours to deliver all the boxes. Linear programming provides the mathematical equations to determine the optimal truck loading and route to be taken in order to meet the requirement of getting all boxes from point A to B with the least amount of going back and forth and, of course, the lowest cost at the fastest time possible.

The basic components of linear programming are as follows:

- Decision variables - These are the quantities to be determined.
- Objective function - This represents how each decision variable would affect the cost, or, simply, the value that needs to be optimized.
- Constraints - These represent how each decision variable would use limited amounts of resources.
- Data - These quantify the relationships between the objective function and the constraints.

Definition

"A mathematical method to allocate scarce resources to competing activities in an optimal manner when the problem can be expressed using a linear objective function and linear inequality constraints."

Linear Equations

All of the equations and inequalities in a linear program must, by definition, be linear. A linear function has the following form:

$a0 + a1\ x1 + a2\ x2 + a3\ x3 + \ldots + an\ xn = 0$

In general, the *a*'s are called the *coefficients* of the equation; they are also sometimes called *parameters*. The important thing to know about the coefficients is that they are fixed values, based on the underlying nature of the problem being solved. The *x*'s are called the *variables* of the equation; they are allowed to take on a range of values within the limits defined by the constraints.

The Decision Variables

The variables in a linear program are a set of quantities that need to be determined in order to solve the problem; i.e., the problem is solved when the best values of the variables have been identified. The variables are sometimes called decision variables because the problem is to decide what value each variable should take. Typically, the variables represent the amount of a resource to use or the level of some activity. For example, a variable might represent the number of acres to cut from a particular part of the forest during a given period.

The Objective Function

The objective of a linear programming problem will be to maximize or to minimize some numerical value. This value may be the expected net present value of a project or a forest property; or it may be the cost of a project; it could also be the amount of wood produced, the expected number of visitor-days at a park, the number of endangered species that will be saved, or the amount of a particular type of habitat to be maintained. The objective function indicates how each variable contributes to the value to be optimized in solving the problem.

The Constraints

Constraints define the possible values that the variables of a linear programming problem may take. They typically represent resource constraints, or the minimum or maximum level of some activity or condition.

Application of Linear Programming

Food and Agriculture

Farmers apply linear programming techniques to their work. By determining what crops they should grow, the quantity of it and how to use it efficiently, farmers can increase their revenue. In nutrition, linear programming provides a powerful tool to aid in planning for dietary needs. In order to provide healthy, low-cost food baskets for needy families, nutritionists can use linear programming. Constraints may include dietary guidelines, nutrient guidance, cultural acceptability or some combination thereof. Mathematical modeling provides assistance to calculate the foods needed to provide nutrition at low cost, in order to prevent non-communicable disease. Unprocessed food data and prices are needed for such calculations; all while respecting the cultural aspects of the food types. The objective function is the total cost of the food basket. Linear programming also allows time variations for the frequency of making such food baskets.

Health care or medical field

The health care field, including doctors and nurses, often use linear equations to calculate medical doses. Linear equations are also used to determine how different medications may interact with each other and how to determine correct dosage amounts to prevent overdose with patients using multiple medications. Doctors also use linear equations to calculate doses based on a patient's age and weight.

Applications in Engineering

Engineers also use linear programming to help solve design and manufacturing problems. For example, in airfoil meshes, engineers seek aerodynamic shape optimization. This allows for the reduction of the drag coefficient of the airfoil. Constraints may include lift coefficient, relative maximum thickness, nose radius and trailing edge angle. Shape optimization seeks to make a shock-free airfoil with a feasible shape. Linear programming therefore provides engineers with an essential tool in shape optimization.

Transportation Optimization

Transportation systems rely upon linear programming for cost and time efficiency. Bus and train routes must factor in scheduling, travel time and passengers. Airlines use linear programming to optimize their profits according to different seat prices and customer demand. Airlines also use linear programming for pilot scheduling and routes. Optimization via linear programming increases airlines' efficiency and decreases expenses.

Energy Industry

Modern energy grid systems incorporate not only traditional electrical systems, but also renewable such as wind and solar photovoltaic. In order to optimize the electric load requirements, generators, transmission and distribution lines, and storage must be taken into account. At the same time, costs must remain sustainable for profits. Linear programming provides a method to optimize the electric power system design. It allows for matching the electric load in the shortest total distance between generation of the electricity and its demand over time. Linear programming can be used to optimize load-matching or to optimize cost, providing a valuable tool to the energy industry.

Architect and Builder

The construction field frequently uses linear equations when measuring and cutting all types of materials for job sites. Both carpenters and electricians are included in the construction field and use linear equations on many of the jobs they do. A carpenter might, for example, use a linear equation to estimate the cost of wood and nails for a remodeling project.

Research Scientists

Scientists of all types use linear equations on a regular basis. Life, physical and social scientists all have situations where linear equations make their jobs easier. Biologists to chemists all use the same linear equation format to solve problems such as determining ingredient portions, sizes of forests and atmospheric conditions. A pharmacist might, for example, set up several linear equations to find the right combination of chemicals needed for an experiment of a medicine.

Financial Analyst

Financial occupations often require the use of linear equations. Accountants, auditors, budget analysts, insurance underwriters and loan officers frequently use linear equations to balance accounts, determine pricing and set budgets. Linear equations used in financial occupations may also be used in creating family budgets as well. A financial planner, for example, uses linear equations to determine the total worth of a client's stocks.

Applications of linear programming for solving business problems:

Business Management

Business Management required to use linear equations in a variety of fields to calculate measurements, make purchases, evaluate raises and determine how many employees are required to complete specific jobs. Some of the more common managerial positions using linear equations include advertising, real estate, purchasing and agriculture. For example, an advertising manager might plan an online ad campaign budget using linear equations based on the cost per click. Human resources positions and even store clerks may find the need for linear equations. This is most common when calculating payroll and purchases without calculators. Linear equations are also used when placing orders for supplies and products, and can help find the lowest costs for an order, taking into account prices and volume discounts.

Efficient Manufacturing

Manufacturing requires transforming raw materials into products that maximize company revenue. Each step of the manufacturing process must work efficiently to reach that goal. For example, raw materials must past through various machines for set amounts of time in an assembly line. To maximize profit, a company can use a linear expression of how much raw material to use. Constraints include the time spent on each machine. Any machines creating bottlenecks must be addressed. The amount of products made may be affected, in order to maximize profit based on the raw materials and the time needed.

a. Production Management:

LP is applied for determining the optimal allocation of such resources as materials, machines, manpower, etc. by a firm. It is used to determine the optimal product- mix of the firm to maximize its revenue. It is also used for product smoothing and assembly line balancing.

b. Personnel Management:

LP technique enables the personnel manager to solve problems relating to recruitment, selection, training, and deployment of manpower to different departments of the firm. It is also used to determine the minimum number of employees required in various shifts to meet production schedule within a time schedule.

c. Inventory Management:

A firm is faced with the problem of inventory management of raw materials and finished products. The objective function in inventory management is to minimise inventory cost and the constraints are space and demand for the product. LP technique is used to solve this problem.

d. Marketing Management:

LP technique enables the marketing manager in analysing the audience coverage of advertising based on the available media, given the advertising budget as the constraint. It also helps the sales executive of a firm in finding the shortest route for his tour. With its use, the marketing manager determines the optimal distribution schedule for transporting the product from different warehouses to various market locations in such a manner that the total transport cost is the minimum.

e. Financial Management:

The financial manager of a firm, mutual fund, insurance company, bank etc. uses the LP technique for the selection of investment portfolio of shares, bonds, etc so as to maximize return on investment.